

A Structural Model of Default Risk *

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Abstract

We price corporate debt from a structural model of firm default. We assume that the capital market brings about efficient firm default when the continuation value of the firm falls below the value it would have after bankruptcy restructuring. This characterization of default makes the model more tractable and parsimonious than the existing structural models. The model can be applied in conjunction with a broad range of default-free interest rate models to price corporate bonds. Closed-form corporate bond prices are derived for various parametric examples. The term structures of yield spreads and durations predicted by our model are consistent with the empirical literature. We

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illustrate the empirical performance of the model by pricing selected corporate bonds with varied credit ratings.

Traditionally the credit risk literature has taken two approaches to the valuation of corporate debt. The ‘structural’ approach models the bankruptcy process explicitly. It defines both the event that triggers default and the payoffs to the bond holders at default in terms of the assets and liabilities of the firm. Although substantial abstraction of the bankruptcy game is required to retain tractability. The ‘reduced-form’, or ‘statistical’ approach, abstracts away completely from the economic notion of bankruptcy and treats default as an event governed by an exogenously specified jump process.¹

The statistical approach is very tractable and has been successful in pricing default risk. Duffie and Singleton (1999) show that any default-free term structure model can be used to price bonds with default risk. One simply models the spot interest rate to include an instantaneous default spread. Affine term structure models can then be tweaked to produce closed-form corporate bond prices.

In contrast, the structural approach of firm default has only been able to produce closed-form prices under extremely simplistic capital structure assumptions. The tradeoff of realism for tractability in structural models has, so far, generated less than satisfactory empirical pricing performances. As a consequence, the applied literature has favored statistical models over structural models. However, beyond good in-sample fit, ultimately, we are interested in linking the determinants of default to firm characteristics. For this purpose, the reduced-form approach is less suitable. We must appeal to the structural models.

¹Since this paper is structural in nature, we refer readers interested in the statistical models to Litterman and Iben (1991), Madan and Unal (1993), Fons (1994), Das and Tufano (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1997, 1998), and Duffie and Singleton (1999), and Duffie and Lando (2001) amongst other excellent treatments on the topic.

Our paper offers a highly tractable structural model of default which performs well empirically. Like other papers in the structural literature, we characterize default as the first time the firm value V crosses a default boundary K . This approach begins with Black and Scholes (1973), Merton (1974) and Black and Cox (1976), continues with Longstaff and Schwartz (1995) and, more recently, with Briys and Varenne (1997), Taurén (1999) and Collin-Dufresne and Goldstein (2001).

In Black and Scholes (1973) and Merton (1974), all debts mature on the same day, and the firm defaults when its value is lower than the payment due. Hence, the default boundary K consists of a single point, equal to the face value of the maturing debt. If default occurs, the various classes of claimants receive the liquidation value of the firm in their order of priorities. Unfortunately, the model becomes intractable when debt obligations mature at various points in time.

Black and Cox (1976) and Longstaff and Schwartz (1995) assume that the firm is forced into default by its debt covenants the first time its value falls below a constant threshold K .² In this case, K can be viewed as the face value of the liabilities of a firm that has a constant dollar amount of debt outstanding at all times. Here, the default boundary is continuous and deterministic (actually constant) so that default may occur at any point in time, even when no payment is due. Black and Cox (1976) model the default payoffs in manners identical to that of Black and Scholes (1973), which again makes the model intractable for realistic capital structures. Longstaff and Schwartz (1995) circumvent the problem through an innovative way of dealing with default payoffs. At default, the corporate bond is exchanged for a

²Black and Cox (1976) actually consider a default boundary of the form $K(t) = ke^{-c(T-t)}$, where T is the time to maturity and k is the face value of the maturing debt.

fraction $(1 - W)$ of an otherwise equivalent default-free bond, where W may depend on the priority and the maturity of the bond.³ This allows the treatment of corporate coupon bonds as a portfolio of corporate zeros. More importantly, it allows the pricing of a corporate bond without the detailed specifications of the rest of the firm's capital structure. Along a similar vein, Briys and Varenne (1997) extends the default boundary of Black and Cox (1976) to allow for stochastic default-free interest rate while adopting the default write-down treatment of Longstaff and Schwartz.⁴

In both Black and Cox (1976), Longstaff and Schwartz (1995) and Briys and Varenne (1997), the debt issued by the firm is assumed to remain constant irrespective of the firm value. This suggests an unreasonable waste of the firm's debt capacity as the firm grows in value. Taurén (1999) and Collin-Dufresne and Goldstein (2001) realize that in reality the dollar amount of the firm's liabilities does not remain constant. They propose alternative models to reflect the firm's tendency to maintain a stationary leverage ratio. Following the interpretation that K is the face value of the firm's liabilities, the ratio V/K can be interpreted as the inverse debt ratio. V/K is then modeled as mean-reverting, and the firm is assumed to enter into default when V/K falls 'dangerously low'. However, Taurén (1999) concedes that empirical estimation with his model is almost impossible due to the computational intensity required.⁵

³This assumption is consistent with empirical studies (Franks and Torous (1989), Eberhart, Moore, and Roenfeldt (1990), Weiss (1990), and Betket (1995)), which suggest that priority rules are almost always violated, with junior claimants receiving payments even when senior claimants are not paid in full.

⁴Briys and Varenne (1997) model the boundary as $K(t) = k\alpha \exp(\int_t^T r(u)du)$, where k is the face value of the debt maturing at time T

⁵Many parameters in Taurén (1999) could not be estimated directly from corporate bond prices but must

Our model, by comparison, is both more realistic and more parsimonious. The innovation in our model comes from the characterization of firm default. This characterization imposes strong restrictions on the default boundary. The crucial realization is that in an efficient capital market, default occurs when the continuation value of the firm under the current management and capital structure is less than the value that the firm would have after bankruptcy. Specifically, bankruptcy can result in a Chapter 7 liquidation, a Chapter 11 reorganization, a Chapter 11 liquidation, or a private debt restructuring. We assume that the bankruptcy code and corporate governance mechanism, coupled with the market for corporate control, ensure that bankruptcy occurs efficiently.

We define the default boundary K as the ‘bankrupt’ firm value (recall that the bankrupt firm value can be either the value of the firm’s liquidated assets or a recapitalized going concern). Market efficiency then predicts default to occur when the firm’s continuation value V falls below K . This characterization of the default boundary necessarily makes K stochastic and covary with V . Additionally, since K represents the value of an asset, its risk-adjusted return can be modeled to equal the default-free interest rate.

We follow Longstaff and Schwartz (1995) in assuming that at default a corporate bond is exchanged for an equivalent default-free bond at a write-down W , which depends on the bond’s priority and maturity. As we mentioned, this feature allows the valuation of each debt issue independent of the rest of the firm’s capital structure.

Our model of default can be coupled with virtually any model of the default-free term structure to price corporate bonds. In particular, when the default-free term structure has be separately assumed or calibrated. The high dimensionality of the estimation also makes it infeasible to estimate more than one zero coupon corporate bond in his paper.

non-stochastic volatility, we are able to derive approximate analytical bond prices. The approximate analytic pricing solution is rapidly convergent and achieves high accuracy with only second order expansion terms. This reduces the computational intensity of the model estimation substantially. For other default-free term structure assumptions, we provide simple numerical methods for computing bond prices.

The remainder of this article is organized as follows. Section I develops the modeling framework. Section II derives closed-form solutions for risky corporate bonds under two particular parameterizations of the model. Also presented are analytical approximate solutions and simulation methods for more general model specifications. Section III discusses the term structure of yield spreads and durations predicted by our model. Section IV examines the empirical performance and illustrates applications of the valuation method. Section V concludes the article.

I. The Valuation Framework

In this section, we present the assumptions about the firm value dynamics and the default boundary dynamics. We make the standard technical assumptions needed to employ risk neutral pricing techniques. In addition, we assume that two sources of risk – shocks to the firm fundamentals and shocks to the default-free interest rate – drive the variations in the continuation and the bankruptcy firm values. The shocks to the firm fundamentals under the risk neutral space Q are characterized by the Brownian motion Z_v , while the shocks to the default-free interest rates are characterized by the Brownian motion Z_r . The instantaneous correlations between dZ_v and dZ_r is $\rho_{rv}dt$.

A. The Continuation Value Dynamics

We define V as the *continuation value* of the firm. It refers to the value of the firm without entering into bankruptcy restructuring. This will be distinguished from the *bankruptcy value* K , which refers to the value that the firm would have after the bankruptcy proceeding. We introduce the dynamics of V first. In the risk-neutral space, the instantaneous total return of V under the Q measure must equal the default-free short rate r . We also make the additional restrictions of constant volatilities. Summarizing, the risk-neutral dynamics of V are governed by:

$$\frac{dV(t)}{V(t)} = [r(t) - \delta_e(t, V, K, r) - \delta_d(t, V, K, r)]dt + \gamma_v dZ_v(t) + \gamma_r dZ_r(t) \quad (1)$$

where $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ are the pay out rates to the equity and the debt holders and can be arbitrary functions of t , V , K and r . γ_v and γ_r are loadings on the shocks to firm fundamentals (dZ_v) and shocks to the default-free short rate (dZ_r). Note, $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ can reflect complicated dividend and debt servicing policies. $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ may be negative to reflect additional equity or debt issuing.

B. The Bankruptcy Value Dynamics

We wish to characterize the default boundary to reflect efficient bankruptcies. We assume that the capital market is efficient, which suggests that firm default occurs when the continuation value of the firm falls below the value the firm would have if it enters bankruptcy restructuring. The institutional features and mechanisms of the financial market such as bankruptcy codes, debt covenants, and the market for corporate control are suppose

to bring about efficient defaulting.

We define the default boundary $K(t)$ as the time t bankruptcy value of the firm. Efficiency then predicts that default occurs the first time $V(t)$ falls below $K(t)$. We now make exact the definition of $K(t)$. For simplicity of exposition, suppose one specific plan of bankruptcy restructuring is available for each firm. For some firms, if default occurs, bankruptcy results in a Chapter 7 or Chapter 11 liquidation. Suppose that the firm faces certain liquidation in the event of default. $K(t)$ then represents the sum value of the physical assets of the firm. Prior to default, the continuation value of the firm is greater than the value of the assets in a piecemeal sell off. However, at default, when $V(t)$ hits $K(t)$ for the first time, the firm is liquidated to generate $K(t)$, which is then distributed to the claimants of the firm. For other firms, bankruptcy results in a Chapter 11 reorganization. Suppose the reorganization plan liquidates the non-cash generating long-term investments and retains only the cash generating assets. $K(t)$ then represents the value of this stripped down company at t . When $V(t)$ hits $K(t)$, the firm defaults and is reorganized, and the claimants receive securities of this new entity.

So defined, K represents an asset value. Therefore, under Q , K 's instantaneous total return must be equal to r . Again, we make the additional restriction of constant volatility on the K process and write the dynamics of K under Q as:

$$\frac{dK(t)}{K(t)} = [r(t) - \delta_e(t, V, K, r) - \delta_d(t, V, K, r)]dt + \beta_v dZ_v(t) + \beta_r dZ_r(t) \quad (2)$$

where γ_v and γ_r are loadings on the shocks to firm fundamentals (dZ_v) and shocks to the default-free short rate (dZ_r).

We argue before that $K(t)$ represents the value of the firm in a piecemeal liquidation if

the firm has no economic value as a going concern. Under this scenario, $K(t)$, for estimation purposes, might be proxied by the book value of the firm's asset. The ratio V/K can then be crudely interpreted as Tobin's q . A low V/K would indicate low economic value added. Financial distress and economic distress therefore occur at the same time. However, when the firm is expected to continue as a going concern in the event of a bankruptcy, V/K does not have the interpretation of a Tobin's q anymore. Financial distress is then a corporate control mechanism which forces reorganization of resources to deliver higher economic value added and can occur without obvious signs of economic distress.

It is important to stress that bankruptcy occurs when the firm's asset is insufficient to cover its liabilities. Our model is consistent with that definition of bankruptcy though it does not explicitly model the firm's liabilities. Note that $K(t)$ is not modeled as the outstanding debt of the firm as is done so in the traditional structural literature. However, $K(t)$ is nonetheless related to the firm's outstanding debt. Recall that the creditors receive a fraction of the bankruptcy value $K(\tau)$ when the firm defaults at time $\tau > t$. Naturally, the firm's ability to finance its operations with debt (or its debt capacity) must be closely related to $K(t)$, which is the conditional expected discounted value of $K(\tau)$; this linkage is substantiated in Williamson (1988) and Shleifer and Vishny (1992). We expect a decrease in $K(t)$ to lead to a reduction in the firm's debt capacity, which forces the firm to substitute some debt financing with equity financing. Similarly, we expect an increase in $K(t)$ to increase debt capacity, which encourages the firm to substitute some equity financing with debt financing. Consequently, when $K(\tau)$ increases or decreases without a proportional movement in $V(t)$, we expect the debt ratio to also move in the same direction.

C. The Solvency Ratio

We now define a new variable, the log-solvency ratio:

$$X(t) \equiv \log \frac{V(t)}{K(t)}. \quad (3)$$

We note that when $X(t)$ hits 0 for the first time, the firm enters into default. Restating default this way avoids having to keep track of both $V(t)$ and $K(t)$. The evolution of $X(t)$ completely describes the default probability.

The dynamics of $X(t)$ are given by:

$$dX(t) = \mu_X dt + \sigma_X dZ_X(t) \quad (4)$$

where Z_X is a new Brownian motion defined by:

$$\sigma_X Z_X(t) = (\gamma_r - \beta_r) Z_r(t) + (\gamma_v - \beta_v) Z_v(t) \quad (5)$$

$$(\sigma_X dZ_X)^2 = [(\gamma_r - \beta_r)^2 + (\gamma_v - \beta_v)^2 + 2\rho_{rv}(\gamma_r - \beta_r)(\gamma_v - \beta_v)] dt \quad (6)$$

and where the drift coefficient is given by:

$$\mu_X = \frac{1}{2} (\sigma_K^2 - \sigma_V^2) \quad (7)$$

where σ_K is the volatility of the bankruptcy value process:

$$\sigma_K^2 = \beta_r^2 + \beta_v^2 + 2\rho_{rv}\beta_r\beta_v \quad (8)$$

and where σ_V is the volatility of the continuation firm value process:

$$\sigma_V^2 = \gamma_r^2 + \gamma_v^2 + 2\rho_{rv}\gamma_r\gamma_v \quad (9)$$

Note, that dZ_X^Q is correlated with dZ_r^Q . The instantaneous correlation coefficient is given by:

$$\rho_{Xr} dt = \frac{\rho_{rv}(\gamma_v - \beta_v) + (\gamma_r - \beta_r)}{\sigma_X} dt. \quad (10)$$

We can now define the condition of default in terms of X . Default is defined to occur at τ , where τ is the first time X hits zero. Note that default does not depend on $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$. This arises because the firm's dividend and debt interest payments impact both the firm value V and the bankrupt value K equally. This feature is distinctively different from other structural models with the exception of stationary leverage models.

D. Debt Write Downs when Default Occurs

We assume that debt restructuring occurs simultaneously for all debt issues once the firm defaults. This is in accordance with cross default provisions that are widely adopted in practice. Next, following Longstaff and Schwartz (1995), we assume that, at default, the holder of a corporate coupon bond receives $1 - W$ of an otherwise equivalent Treasury bond; where the write down W is sometimes referred to by ratings agencies as the loss severity (see Cantor and Fons (1999)), and where W is larger for bonds with lower priorities. For example, consider two corporate bonds – a senior secured and a junior note. The write down W_1 for the senior note would be less than the write down W_2 for the junior note. For modeling simplicity W is assumed deterministic; equivalently, we may assume that W is stochastic but uncorrelated with other stochastic processes in the model.

Since a corporate coupon bond, in default, is exchanged for an equivalent Treasury bond, which is a portfolio of Treasury zeros, it can be replicated by a portfolio of corporate zeros in an obvious way. The firm's outstanding debt issues can then be modeled as a collection of corporate zeros that are defaulted on at the same time, and each corporate zero coupon bond can be priced independent of the other zeros. This formulation allows each corporate bond to be priced independently of the firm's other liabilities, making the detailed

description of the firm's capital structure unnecessary for the pricing of the corporate bonds!

We write the time t value of a corporate zero coupon bond with maturity T as:

$$\begin{aligned} C(t, T) &= \mathbb{E}_t^Q[(1 - W\mathbf{1}(\{\tau \leq T\}))e^{-\int_t^T r(u)du}] \\ &= P(t, T) - W\mathbb{E}_t^Q[\mathbf{1}(\{\tau \leq T\})e^{-\int_t^T r(u)du}] \end{aligned} \quad (11)$$

where $P(t, T)$ is the price of the T -maturity Treasury zero, τ is the first time X hits zero, $\mathbf{1}(\{\tau \leq T\})$ is an indicator function which takes on the value 1 if $\tau \leq T$, and the expectation is taken over Q .

E. The Short Rate Dynamics

We now specify the dynamics of the default-free term structure. We specify a general process for the default-free short rate first. Particular parameterizations of the process are introduced later.

The dynamics of the instantaneous default-free interest rate r are governed by:

$$dr(t) = \mu_r(r, t)dt + \sigma_r(r, t)dZ_r(t) \quad (12)$$

where $\mu_r(r, t)$ and $\sigma_r(r, t)$ can be functions of r and t (and are left unspecified for now), and $Z_r(t)$ is the same Brownian motion that shocks V and K . The introduction of stochastic default-free interest rates is important for examining the impact of interest rate risk on the default probability and for explaining the observed differences in credit spreads for firms with similar credit ratings.

II. Valuation of Corporate Debt Securities

Examining (11), we need to specify a model of the term structure of default-free interest rate to compute the pure discount bond price $P(t, T)$. We need to further characterize the first hitting time τ to evaluate the expectation $E_t^Q[\mathbf{1}(\{\tau \leq T\}) \exp(-\int_t^T r(u)du)]$. We postpone specifying the interest rate process until later since our framework is compatible with most commonly used default-free term structure models.

A. Bond Pricing Formula Under the Forward Measure Q_T

As is often true, using the discount bond with price $P(t, T)$ as the numeraire simplifies the algebra. We re-write the formula for a corporate discount bond in (11) as:

$$\begin{aligned}
 C(t, T) &= P(t, T)E_t^{Q_T}[(1 - W\mathbf{1}(\{\tau \leq T\})] \\
 &= P(t, T)(1 - WE_t^{Q_T}[\mathbf{1}(\{\tau \leq T\})]) \\
 &= P(t, T)(1 - W\Pi(t, T))
 \end{aligned} \tag{13}$$

where the expectation is taken under the forward measure Q_T and where we define $\Pi(t, T)$ (the forward default probability) as the probability that default occurs between t and T under Q_T .⁶ This formulation of the bond prices is easier to work with.

To make use of equation (13) to price corporate bonds, we need to re-express the dynamics of X under Q_T :

$$dX(t) = (\mu_X + \rho_{Xr}\sigma_X s(t, T)) dt + \sigma_X dZ_X^{Q_T}(t) \tag{14}$$

where $s(t, T)$ is the volatility of the T-maturity discount bond and $Z_X^{Q_T}$ is a standard Brownian motion under Q_T (see Hsu et. al. for the complete derivation). The drift of X under Q_T

⁶See Baxter and Rennie (1996) or Duffie (1996) for references to the forward probability.

has an additional term $\rho_{Xr}\sigma_X s(t, T)$ that serves to correct for the interest rate risk.

For $\rho_{Xr} > 0$, the shocks to the log-solvency ratio are positively correlated with the shocks to the short rate. So increases in the default probability (decreases in the log-solvency ratio), which reduce corporate bond prices, are likely associated with decreases in the interest rate, which increase corporate bond prices. Intuitively, the two sources of risk partially offset each other, resulting in lower credit spreads than if $\rho_{Xr} \leq 0$

The forward risk adjusted probability of default $\Pi(t, T)$ can, in general, be computed by simulation, although closed-form and analytic approximation solutions are available under more restrictive assumptions. We present these special cases in the next section.

C. Computing Bond Prices

1. Independent Default Risk and Interest Rate Risk

Under specific parameterizations of the firm value process V and the default boundary process K , ρ_{Xr} will be zero, indicating that the default probability is independent of r under Q^T .⁷ An equivalent assumption is made in Jarrow and Turnbull (1995) and is implicit in models where r is non-stochastic. When $\rho_{Xr} = 0$, $\Pi(t, T)$ can be computed in closed-form for any prescribed term structure model; it is simply the probability of an arithmetic Brownian motion, starting from the initial value $X(t)$, with drift μ_X and volatility σ_X , hitting zero before time T .

The first-passage time density of X evaluated at $\tau > t$ is (See Karatzas and Shreve

⁷Recall from (10) that ρ_{Xr} is zero when $\rho_{rv}(\gamma_v - \beta_v) + (\gamma_r - \beta_r) = 0$

(1991)):

$$\phi(\tau) = \frac{X(t)}{\sigma_X(2\pi)^{1/2}(\tau-t)^{3/2}} \exp \left\{ -\frac{[X(t) + \mu_X(\tau-t)]^2}{2\sigma_X^2(\tau-t)} \right\} \quad (15)$$

so that

$$\Pi(t, T) = 1 - \mathcal{N} \left(\frac{X(t) + \mu_X(T-t)}{\sigma_X \sqrt{T-t}} \right) + \exp \left\{ \frac{-2\mu_X X(t)}{\sigma_X^2} \right\} \mathcal{N} \left(\frac{X(t) - \mu_X(T-t)}{\sigma_X \sqrt{T-t}} \right) \quad (16)$$

where \mathcal{N} denotes the standard normal cumulative distribution function.

We note that this model gives closed-form corporate bond prices when coupled with any model of the default-free term structure that produces closed-form Treasury bond prices. We will examine the validity of the assumption that $\rho_{Xr} = 0$ in the empirical portion of the paper.

2. Deterministic Bond Volatilities

We are also able to derive analytical approximate bond prices when the volatility of the T-maturity Treasury bond is a deterministic function of time. Term structure models like Vasicek (1977), Ho and Lee (1986), Hull and White (1990), or other models in the Heath, Jarrow and Morton (1992) framework produce bond prices with deterministic volatilities. With this assumption, the first-passage time problem can be restated as the first-passage time of a standard arithmetic Brownian motion through a deterministic boundary. The formula for the boundary is given by:

$$B(\tau) = \frac{X(t) + \mu_X(\tau-t)}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X} \int_t^\tau s(u, T) du \quad (17)$$

where, again, $s(t, T)$ is the T-maturity Treasury bond return volatility (see Hsu et. al (2004) for the complete derivation).

The default probability Π_t^T can be approximated very efficiently in closed form. Durbin (1992) shows that the first-passage time probability can be approximated to a high degree of accuracy by the following approximation⁸:

$$\begin{aligned} \Pi(t, T) \approx & \int_t^T \left(\frac{B(u)}{u-t} - B'(u) \right) \varphi(u) du \\ & - \int_t^T \int_t^u \left(\frac{B(v)}{v-t} - B'(v) \right) \left(\frac{B(u) - B(v)}{u-v} - B'(u) \right) \varphi(u, v) dv du \end{aligned} \quad (18)$$

where $B'(u)$ denotes the slope of the boundary at u ; $\varphi(u)$ is the density of the Brownian motion at time u , evaluated at $B(u)$; and $\varphi(u, v)$ is the joint density of the Brownian motion at times u and v , evaluated at $B(u)$ and $B(v)$:

$$\varphi(u) = (2\pi(u-t))^{-1/2} \exp \left\{ \frac{-(B(u) - B(t))^2}{2(u-t)} \right\} \quad (19)$$

and

$$\varphi(u, v) = \varphi(u)(2\pi(u-v))^{-1/2} \exp \left\{ \frac{-(B(u) - B(v))^2}{2(u-v)} \right\} \quad (20)$$

The first-passage time probability can be easily computed with simple numeric quadrature methods to evaluate the integrals in equation (18).

3. General Specification

For more general short rate processes, corporate bond prices can still be valued with relative ease through simulation, though the computational intensity increases substantially.

For an arbitrary short rate process under the risk-neutral measure:

$$dr(t) = \mu(r, t)dt + \sigma(r, t)dZ_r(t) \quad (21)$$

⁸Additional expansion terms may be added to further improve accuracy.

we simulate the following processes jointly to compute $\Pi(t, T)$:

$$dr(t) = [\mu(r, t) + \sigma(r, t)s(r, t)] dt + \sigma(r, t)dZ_r^{Q_T}(t) \quad (22)$$

$$dX(t) = [\mu_X + \rho_{Xr}\sigma_X s(r, t)] dt + \rho_{Xr}\sigma_X dZ_r^{Q_T}(t) + \sqrt{1 - \rho_{Xr}^2}\sigma_X dZ_o^{Q_T}(t)$$

where, again, $s(r, t, T)$ is the volatility of the T-maturity Treasury bond, and the two Brownian motions $Z_r^{Q_T}$ and $Z_o^{Q_T}$ are orthogonal by construction. The probability of X hitting 0 before T under Q_T can then be computed simply as the fraction of the paths which reach zero before T .

As a simple illustrative example, consider the one-factor Cox, Ingersoll and Ross (1995) model:

$$dr(t) = a(b - r(t)) dt + \kappa\sqrt{r(t)}dZ_r(t). \quad (23)$$

Bonds can be priced in closed form and have volatilities equal to $s(t, T) = \kappa B(t, T)\sqrt{r(t)}$

where:

$$B(t, T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \quad (24)$$

and $\gamma = \sqrt{a^2 + 2\kappa^2}$.

To compute $\Pi(t, T)$, we only need to simulate jointly the following two processes:

$$dr(t) = (a(b - r(t)) + \kappa^2 B(t, T)r(t)) dt + \kappa\sqrt{r(t)}dZ_r^{Q_T}(t) \quad (25)$$

$$dX(t) = (\mu_X + \rho_{Xr}\sigma_X \kappa B(t, T)\sqrt{r(t)}) dt + \rho_{Xr}\sigma_X dZ_r^{Q_T}(t) + \sqrt{1 - \rho_{Xr}^2}\sigma_X dZ_o^{Q_T}(t)$$

III. The Term Structure of Yield Spreads and Duration

A. Yield Spread

In this section we plot the term structure of yield spreads for varying values of the parameters. We consider our model in conjunction with the Vasicek and the CIR short rate processes. For the Vasicek case, corporate bond prices can be approximated analytically.⁹ For the CIR case, we compute bond prices numerically. The yield spread is computed as

$$\begin{aligned} y(t, T) &= \frac{1}{T-t} \log \frac{P(t, T)}{C(t, T)} \\ &= \frac{1}{T-t} \log \frac{1}{1 - W\Pi(t, T)} \end{aligned} \quad (26)$$

We note that the yield spread depends positively on the write down rate W and the forward default probability $\Pi(t, T)$. Since we can interpret W as a proxy for priority and the forward probability as a proxy for default risk, the model predicts yield spreads to be increasing in default risk and decreasing in priority, which agrees with intuition.

In figure 1, we plot the yield spread for various values of the solvency ratio $X(0)$. We find that less solvent firms display a humped credit spread term structure, while the term structure is monotone increasing for firms with high solvency. This is consistent with the empirical results of Sarig and Warga (1989) and Bohn (1999).¹⁰ In addition, we find that shorter term bonds are more sensitive to changes in the solvency of the issuing firms and the sensitivity is highest for the low solvency (rating) bonds. For a corporate bond of four year maturity, a fall in the solvency ratio from five to three increases the yield spread from 10 to 80 basis points. A fall in solvency ratio from three to two increases the yield spread from 80

⁹See Appendix C for the application of Durbin's rapidly convergent approximation formula for the Vasicek specification

¹⁰Helwege and Turner (1998) find no empirical support for humped credit spread. However, Bohn (1999), using a larger sample of low quality issues, finds strong evidence for humped credit spread.

to 330 basis points. For a corporate bond of twenty year maturity, the similar changes in the solvency ratio raise the credit spread from 60 to 115 to 180 basis points, respectively. Note that as maturity goes to zero, credit spread also goes to zero. This is standard in a perfect information environment where investors observe the firm's asset level. However, in a noisy environment, such as the one described in Duffie and Lando (2001), a firm with asset level lower than its book liability may be able to raise additional debt capital. In which case a credit spread would exist even for a debt of zero maturity. Huang and Huang (2002) find empirical evidence that yield spread does not shrink to zero with decreasing maturity. This empirical observation cannot be reconciled with any of the known structural models and is a defect of this literature. We investigate in the empirical section the degree to which this issue impacts our model's pricing performance.

The intuition for the humped credit spread term structure is clear. For very low grade bonds, the probability of default does not increase dramatically with maturity beyond the first few years. The prediction of a decreasing conditional probability of default and the associated humped-shape yield spread term structure for low solvency firms are, however, absent in the statistical models. Traditional statistical models assume instead a constant conditional default probability, which is appropriate for the examination of the swap spread or generic corporate spread for a portfolio of bonds belonging to a given credit class, but is less well-suited for studying individual firm default. Fons (2002) reports that annually, on average, 25% of all rated corporate bonds migrate to different credit categories, suggesting that for the average firm, $X(t)$ is rather volatile.¹¹

¹¹The average yield difference between a Moody Baa and A with a maturity of 20 years is 48 basis points for the last 20 years. (Source: Moody's Investor Service.)

In figure 2, we see that the yield spread is decreasing with μ_X . This is obvious. We expect μ_X to be small in magnitude relative to the level of X for a firm not near bankruptcy. A large negative μ_X would indicate a capital structure policy, which leads quickly to bankruptcy – a scenario that appears unlikely. In the empirical section, we see that the estimated μ_X is small relative to the level of X (usually 1/100th the value of X) and is typically insignificantly different from zero. Contrasting to models with constant default boundaries, where the log-solvency ratio has an assumed positive drift, our model suggests a higher yield spread for corporate bonds.

In figure 3, we plot the term structure of yield spreads for various values of ρ_{Xr} . We see that the yield spread is decreasing in ρ_{Xr} . This observation has important implications for the yield spreads paid by counter-cyclical firms ($\rho_{Xr} < 0$) and cyclical firms ($\rho_{Xr} > 0$) which have otherwise identical credit ratings. For $\rho_{Xr} > 0$, the firm's default probability increases (or X decreases) when interest rate decreases. The former effect decreases the corporate bond price while the latter increases it, creating offsetting effects. It is worth noting that ρ_{Xr} impacts the default probability substantially in our static comparison, and the effect is intensified by both maturity and solvency. The difference in yields between the following two sets of parameters $\{X(0) = \log(2), \rho_{Xr} = 0\}$ and $\{X(0) = \log(2), \rho_{Xr} = 0.15\}$ at 4 year maturity is around 35 bps, while the difference between $\{X(0) = \log(5), \rho_{Xr} = 0\}$ and $\{X(0) = \log(5), \rho_{Xr} = 0.15\}$ at the same maturity is less than 5 bps. So a less solvent firm suffers a larger increase in yield spread when ρ_{Xr} decreases. However, the yield difference between $\{X(0) = \log(5), \rho_{Xr} = 0\}$ and $\{X(0) = \log(5), \rho_{Xr} = 0.15\}$ at 20 year maturity increases to around 40 bps. So a longer maturity corporate debt suffers a larger increase in yield spread when ρ_{Xr} decreases.

In figure 4, we see that the yield spread is increasing with σ_X . The more volatile is the firm's log-solvency ratio, the more likely default occurs. From equation (6) we know that:

$$\sigma_X^2 = (\gamma_r - \beta_r)^2 + (\gamma_v - \beta_v)^2 + 2\rho_{rv}(\gamma_r - \beta_r)(\gamma_v - \beta_v). \quad (27)$$

Therefore, the volatility of the solvency ratio depends most importantly on the relative responses of V and K to the firm fundamental shocks and the interest rate shocks. If V and K respond in near tandem to the two shocks, then σ_X is small and vice versa. We note that for a growth firm, V is likely to be substantially more volatile than K , suggesting a high cost of debt financing even if X is high.

In figure 5, we plot the yield spread for various values of the write down ratio W . The yield spread increases with W as expected. The greater the loss of value to the principal during the debt renegotiation, the greater the risk premium demanded up front. We note, however, that since W does not impact the default probability; it also does not impact the curvature of the yield spread. Thus, insofar as yield spreads and credit ratings issued by rating agencies reflect both the default probabilities and debt write downs, our model can disjoin the two effects easily by examining the shape and the level of the yield spread. For statistical models, disjoining the effects of the write down ratio W from the default intensity is impossible. The ability to estimate the write down ratio provides some real advantages. Traditionally, the recovery rates $(1 - W)$ for different debt priority classes can only be estimated from bond issues that have been defaulted on (see Altman (1992), Frank and Torous (1994), and Fons (2002) for studies on recovery rates for different priority and rating classes). This severely limits the size of the sample. With our model, we can estimate the recovery rates using all traded debt issues, which significantly increase the size of the

sample.

B. Duration

We now turn our attention to the duration of corporate bonds. We can write the duration of the corporate discount bond as:

$$\frac{\partial C(t, T)/\partial r}{C(t, T)} = -\frac{\partial P(t, T)/\partial r}{P(t, T)} - \frac{\partial(1 - W\Pi_t^T)/\partial r}{(1 - W\Pi_t^T)} \quad (28)$$

where it is convenient to interpret $1 - W\Pi(t, T)$ as the *risk and interest rate adjusted* recovery rate on the loan. Bond prices respond to changes in the short rate through two channels. First, the discounted value of the bond's promised cash flows depends critically on r ; this is the Treasury component of the corporate bond. Second, the adjusted recovery rate may also depend on r since the probability of default may depend on r . For the Vasicek model, $\Pi(t, T)$ does not depend on r , so the duration is always equal to the duration of a Treasury bond. For the CIR model, $\Pi(t, T)$ does depend on r . From equation (26), we see that $X(t)$ is increasing in $r(t)$ (or $\Pi(t, T)$ is decreasing in $r(t)$) when $\rho_{Xr} > 0$, and vice versa. Therefore, when $\rho_{Xr} > 0$, the risk and interest rate adjusted recovery rate is increasing in $r(t)$, suggesting a duration for the corporate bond that is lower than the Treasury discount bond, which is consistent with the observation of Chance (1990). However, numerically, the impact of an increase in the instantaneous short rate is non-existent on the forward default probability $\Pi(t, T)$. Numerically, we are unable to produce negative durations within reasonable or even extreme ranges of parameters. So we do not predict negative durations. This prediction distinguishes our model from credit models with constant default boundaries, where negative duration is easily produced for low grade bonds. We consider the prediction of non-negative

durations intuitive and desirable; there is no empirical evidence substantiating the existence of negative duration bonds.

IV. Empirical Analysis

Tractability issue has limited empirical analysis of structural models. To reduce the computation complexity of the estimation problem, model parameters are often calibrated rather than estimated from actual price data. The calibrated model is then used to fit the price data to determine model performance. Using this approach, Jones, Mason and Rosenfeld (1984), Ogden (1987) and Lyden and Saraniti (2000) have found Merton type models to overprice corporate bonds. In an expanded study, Eom, Helwege and Huang (2002) examine five structural models (including also Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001)) and find that Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models is able to deliver higher yield spreads because of the inclusion of stochastic default-free rates; however, the pricing error is large (the average error for the models is more than 100% of the predicted yield spreads), and extremely large or small spreads are common. The non-performance may be attributed partly to the accuracy of the parameter calibration exercise. However, more interesting is the evidence that all five models are found to have particular difficulties pricing bond issues from firms with low leverage ratios and low firm value volatilities; bonds with short durations are also priced with greater errors. In this section we explicitly address the calibration concern by estimating all model parameters with price data. We find that our model produces pricing errors that are the same size as

the bid-ask spreads of the bonds. In addition we do not find difficulties pricing bonds from firms with low leverage ratios and asset volatilities or with low durations.

For our empirical study, we use the parametric model with the Vasicek default-free term structure specification. We note that no essential benefits are gained by adopting the CIR specification, which is computationally much more intensive. Since our focus is on the term structure of the yield spread rather than on the Treasury yield curve, we adopt the more convenient Vasicek specification here.

A. The Data

We estimate the model for corporate bonds issued by nine different issuers with S&P bond ratings ranging from AAA to BB. The empirical exercise is not exhaustive due to the computation intensity of the estimation. We seek instead to illustrate the applicability of the model for various risk classes. Firms represented in this sample are selected to satisfy the following screening criteria.

1. NYSE listed and traded – A numbers of corporate bonds are now listed and traded on the NYSE Automated Bond System. Daily closing prices on these bonds are available from Datastream. In addition, the traded volume as well as the bid-ask spreads are available from the NYSE quote reporting system. This requirement ensures that the reported prices are traded prices rather than soft quotes or matrix inferred prices.
2. Liquid issues – as determined by examining the trading volume and daily price movements.
3. Dollar denominated.
4. Non-callable.

5. No sinking fund requirement.
6. Have at least 5 years of daily data ending in December 1999.
7. Have more than two bond issues in the same claimant class satisfying 1-6.

Bond issuer and issuance information are collected from SDC and cross-referenced against information in Datastream and the S&P Bond Guide. The summary statistics of the selected bond issuers and issues are listed in Table 1.

It is worth noting that the average bid-ask spread (quoted in yields) for the bond issuers in our sample is about 40 basis points. The bid-ask spread data are collected from the NYSE Automated Bond System for all currently traded bonds issued by the 9 firms in our sample. The large bid-ask spread alerts us to micro-structure problems such as return auto-correlation arising from bid-ask bounce. The concern, however, is moderated by using monthly frequency data.

The price data for the selected bonds are obtained from Datastream. To avoid the problem with stale data resulting from non-trading, we create monthly price data from daily data. Specifically, we mark consecutive days of identical prices as stale and then create monthly price data using the first trading day of the month. If the first trading day price is stale, a missing data flag is inserted and the particular data point is discarded in our estimation. Finally, prices are adjusted for accrued interest. For the Treasury rates, we construct the appropriate default-free term structure for each first trading day in the sample using the Vasicek model and the yield information from the corresponding Treasury strips.

B. The Method

The detailed derivation of the econometrics model used in the bond pricing exercise is omitted to improve brevity and readability. Interested readers are encouraged to read Hsu et. al. (2004) for the details on the GMM model and the estimation procedure.

C. The Results

Table 2 presents the parameter estimates and their standard errors for each of the 9 corporate bonds. The row *RMSE* reports the average bond pricing errors in yields for each firm. The average pricing error is 39 basis points. Recall that the average bid-ask spread for the firms in our sample is 41 basis points, so the *RMSE* is quite low compared to the precision of the data.

We report the average solvency ratio V/K and log-solvency ratio \hat{X}_t also. Observe that both the estimated $\hat{\mu}_p$ and $\hat{\mu}_X$ are between 1/10th to 1/100th the size of \hat{X}_t . In fact, we cannot conclude that $\hat{\mu}_p$ and $\hat{\mu}_X$ are different from zero! We expect this to be the case. If the μ 's were large compared to X , then the firm drifts deterministically toward 100 percent equity financing or 100 percent debt financing. This cannot represent a stationary (or even reasonable) capital structure policy.

The estimated $\hat{\rho}_{Xr}$ is significantly different from zero for only 3 out the 9 firms; we have alerted the reader to the difficulty of estimating the non-standardized parameters before. The standardized estimates $\frac{\hat{\rho}_{Xr}}{\sigma_X}$, which are not separately reported here, are all significantly non-zero, suggesting that the interaction between the default-free interest rates and the default probability is important in determining the corporate yield spreads.

In addition, for 7 out of the 9 firms, the estimated $\hat{\rho}_{Xr}$ is positive. A positive ρ_{Xr} indicates that the forward probability of default is negatively correlated with the interest

rate, or that changes in the yield spread are negatively correlated with the changes in the interest rate. This is observed in Longstaff and Schwartz (1995) and confirmed in Taurén (1999) and Duffee (1999).

As we mentioned before, our model allows W and the forward default probability to be estimated separately. Of the 9 issuers in our sample, 7 issuers had junior debts outstanding only. Franks and Torous (1994) and Altman (1992) report that the average write downs for junior debt issues are 0.693 and 0.720 respectively. In our sample, the average write down for junior debt issues is 0.7582. In addition, Franks and Torous (1994) also finds that the average write down for guaranteed issues is 0.395. We have only one issuer with 4 guaranteed notes outstanding, and we estimate a write-down of 0.3852 for those issues. For senior issues, Franks and Torous (1994) find an average write down of 0.530. We have one issuer with 2 senior bonds, and we estimate a write down of 0.4982 for those issues. One useful empirical feature of the model is the estimation of default write downs using bond data from non-defaulting firms. In Altman (1992) and Franks and Torous (1994), the samples include only firms that have filed for bankruptcy protections. The current method, by comparison, can estimate write downs using bond data on solvent issuers, which provides researchers with more cross-sectional observations.

We now present evidence that the model does not show systematic pricing errors with respect to the time to maturity of the corporate bonds or the leverage ratio or the volatility of the firm value. To examine the relationship between the model pricing errors and the time to maturity of the bond, we regress the pricing errors on time to maturity for each bond (33 regressions in total). The average absolute value of the t -statistic for the coefficient is 1.18, suggesting no statistical significant relationship. In our data sample, the shortest time

to maturity was more than two years. Therefore, it is possible that mispricing at the very short maturities suggested by Duffie and Lando (2001) and Huang and Huang (2002) cannot be observed clearly. To address the pricing error bias reported by Eom, Helwege and Huang (2003), we examine the RMSE for each of the nine firm with respect to the volatility of the equity value and the book debt ratio as well as with respect to the volatility and the level of the log-solvency ratio. No discernable relationships are observed.

These results combined with the smaller pricing errors, relative to what have been empirically measured using other structural models, give us confidence that our model can be applied fruitfully in practice when combined with our proposed estimation technique. Specifically, our model can be used to determine the price of a firm's new or infrequently traded bond issues. We can estimate the bond pricing parameters for the issuer by applying the model to its liquidly traded bonds. The estimated parameters can then be used to compute the bond prices of the non-traded or less liquid bond issues that the issuer has outstanding.

From the empirical results, we believe that future structural models should attempt to identify the factors which drive the firm value V and the default boundary value K . In doing so we can place additional restriction on the solvency process X . This will allow us to examine more carefully the relationships between the firm's capital structure evolution and the firm's solvency as well as the costs of the firm's financing.

V. Conclusion

This article presents a structural model of default risk that allows for tractable pricing of corporate fixed rate debts. The model can be specified in conjunction with a wide range of default-free term structure specifications. The theoretical properties of the model are attractive and consistent with the empirical literature on default yield spread and risky bond duration. In addition, the model compares favorably with and offers improvement over the existing structural models. We also estimate the model using panel data of bond prices from 9 firms, and illustrate a relatively fast estimation technique as well as some useful applications of our model. Our estimation technique offers obvious advantages over the calibration techniques that have been applied to study structural models. We find that our model, combined with our GMM estimation, produces low pricing errors and do not suffer from the pricing biases observed by recent empirical studies on existing structural models.

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Table 1: Summary Statistics for the Bond Issuers

	Exxon	ARCO	Eli Lilly	Pacific Bell	IBM	Ford	RJR Nabisco	Safeway	United Airline
Data Range	11/94-12/99	02/90-12/99	02/95-12/99	10/93-12/99	07/93-12/99	11/95-12/99	10/93-12/98	03/95-12/99	08/94-12/99
Bond Issues Selected	4	3	5	2	3	4	5	5	2
Claimant Class	(Guaranteed Notes)	(Debenture)	(Notes)	(Notes)	(Notes)	(Notes)	(Notes)	(Sr. Secured, Sr. Sub.)	(Debenture)
Bid-Ask Spread (% of bond price)	1.66%	1.27%	0.23%	0.38%	2.28%	0.94%	1.25%	1.74%	1.60%
Bid-Ask Spread (yield form [bps])	41.46	31.84	45.01	42.89	34.01	13.45	44.18	52.11	62.70
Average Issue Size (\$mil)	638	417	260	312	850	438	695	185	370
S&P Rating on 12/99	AAA	AA+	AA	AA-	A+	A+	BBB-	BBB, BBB-	BB+
S&P Rating on 12/97	AAA	A	AA	AA-	AA	A	BBB-	BBB, BBB-	BB+
S&P Rating on 12/95	AAA	A	AA	AA-	AA	A+	BBB-	BBB, BB+	BB
S&P Rating on 12/93	--	A+	--	AA-	AA	--	BBB-	--	--
S&P Rating on 12/91	--	A+	--	--	--	--	--	--	--

This table shows the summary statistics for the nine issuers in our sample. Each issuer has at least 60 months of price observations for each of its bonds included in the study. The claimant classes of the bonds are collected from the SDC database. Note that Safeway is the only issuer with bonds from two different claimant classes; 3 of the 5 Safeway bonds are rated senior subordinate, while 2 are rated senior secured. We also report the average bid-ask spread for the issuers in the yield form. This gives us an idea of the quality of the bond price data as well as a reasonable magnitude for the pricing errors. Finally, we report the ratings of the bonds from 1991 through to 1999. In general bonds in the same claimant class from the same issuer have the same rating. For 50% of the cases, we observe rating migration over the life of the bond.

Table 2: Parameter Estimates from GMM Estimation

	Exxon	ARCO	Eli Lilly	Pacific Bell	IBM	Ford	RJR Nabisco	Safeway	United Airline
μ_X	0.011* (0.0095)	-0.017 (0.047)	0.186* (0.189)	0.0247 (0.0355)	-0.0246* (0.0178)	-0.0013* (0.0008)	-0.0576* (0.0266)	0.1134 (0.1364)	0.0642* (0.0355)
$\rho_{X,r}$	0.1568* (0.1357)	0.0399* (0.1076)	0.1518* (0.1541)	0.0839* (0.1209)	0.0624* (0.1028)	-0.291* (0.1439)	0.2482* (0.1145)	0.1177* (0.1415)	-0.1631* (0.0886)
σ_X	0.3196 (0.2765)	0.057 (0.1536)	0.5998 (0.6088)	0.1288 (0.185)	0.1811 (0.32)	0.4585 (0.2267)	0.4710 (0.2174)	0.7449 (0.8959)	0.3171 (0.1737)
W	0.3852 (0.098)	0.6784 (0.020)	0.7968 (0.026)	0.7376 (0.0253)	0.6821 (0.0909)	0.7079 (0.029)	0.7458 (0.120)	0.498 ¹ , 0.328 ² (0.078), (0.040)	0.8827 (0.0321)
μ_P	-0.0797 (0.2228)	-0.0039 (0.0429)	-0.0579 (0.4301)	-0.0024 (0.0801)	0.03565 (0.04273)	-0.142 (0.3362)	0.0146 (0.2887)	0.0127 (0.4977)	0.0513 (0.208)
RMSE (in yields bps)	26.84	43.99	49.05	29.76	47.56	37.64	41.61	43.53	27.08
Average(log(V/K))	1.24	0.37	1.92	0.40	0.23	1.71	1.61	2.02	1.08
Average(V/K)	3.46	1.44	6.85	1.49	1.26	5.54	5.02	7.55	2.93

This table shows the GMM estimates for our model. The reported estimates for $\mu_X, \mu_P, \rho_{X,r}, \text{and } \sigma_X$ are annualized. The standard errors are reported in the parentheses. The * signifies that the standardize estimate is significant at the 5% level. The pricing error for the bonds from each issuer is reported in the row RMSE. Note that these numbers are of the same magnitude as the bid-ask spread reported in Table 1.

Yield Spread for Different Values of X

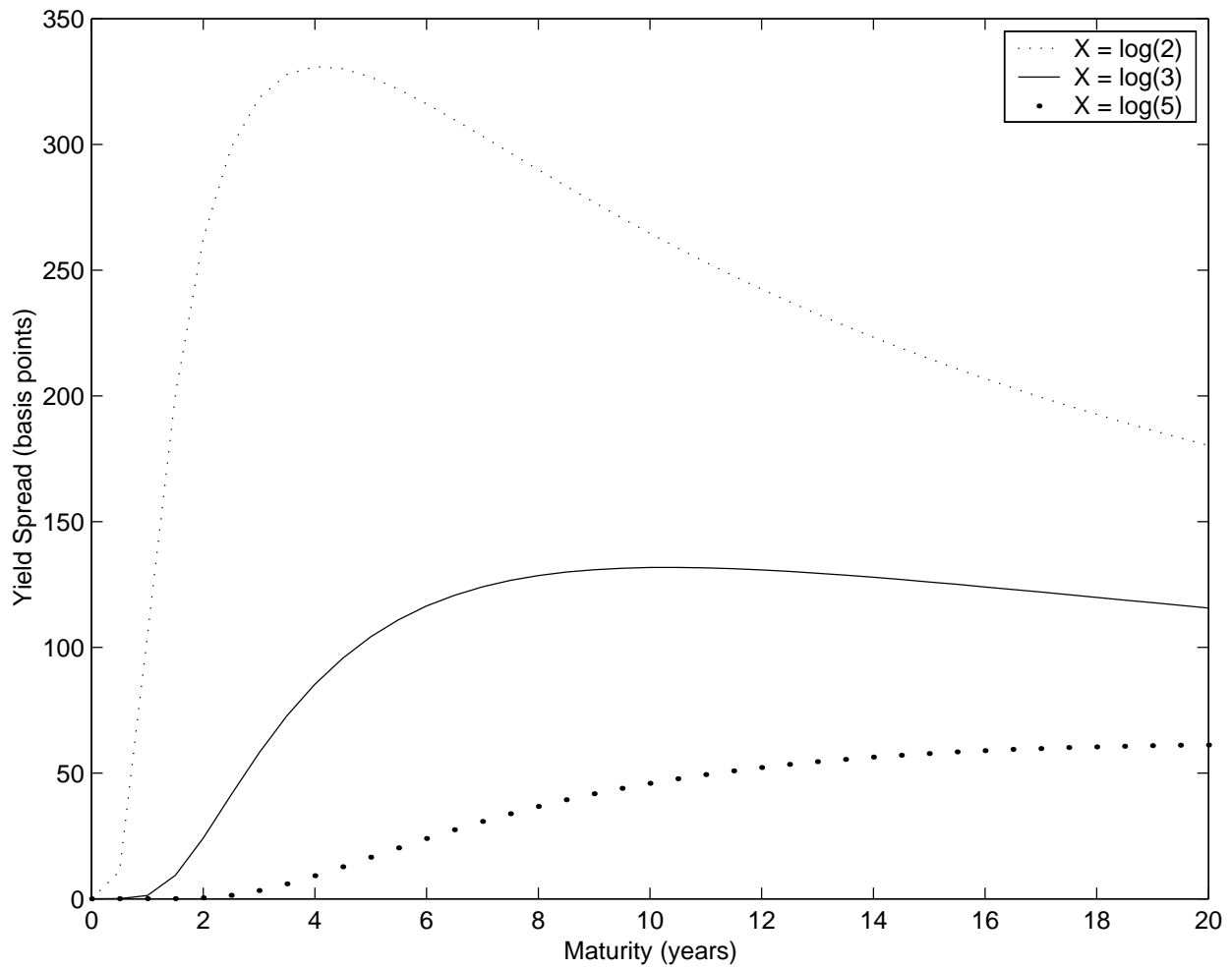


Figure 1. This figure plots the term structure of yield spread for different values of the log-solvency ratio X . The other parameters of the model are held fixed at $\mu_X = 0$, $\rho_{Xr} = 0$, $\sigma_X = 0.3$, and $W = 0.5$.

Yield Spread for Different Values of μ_X

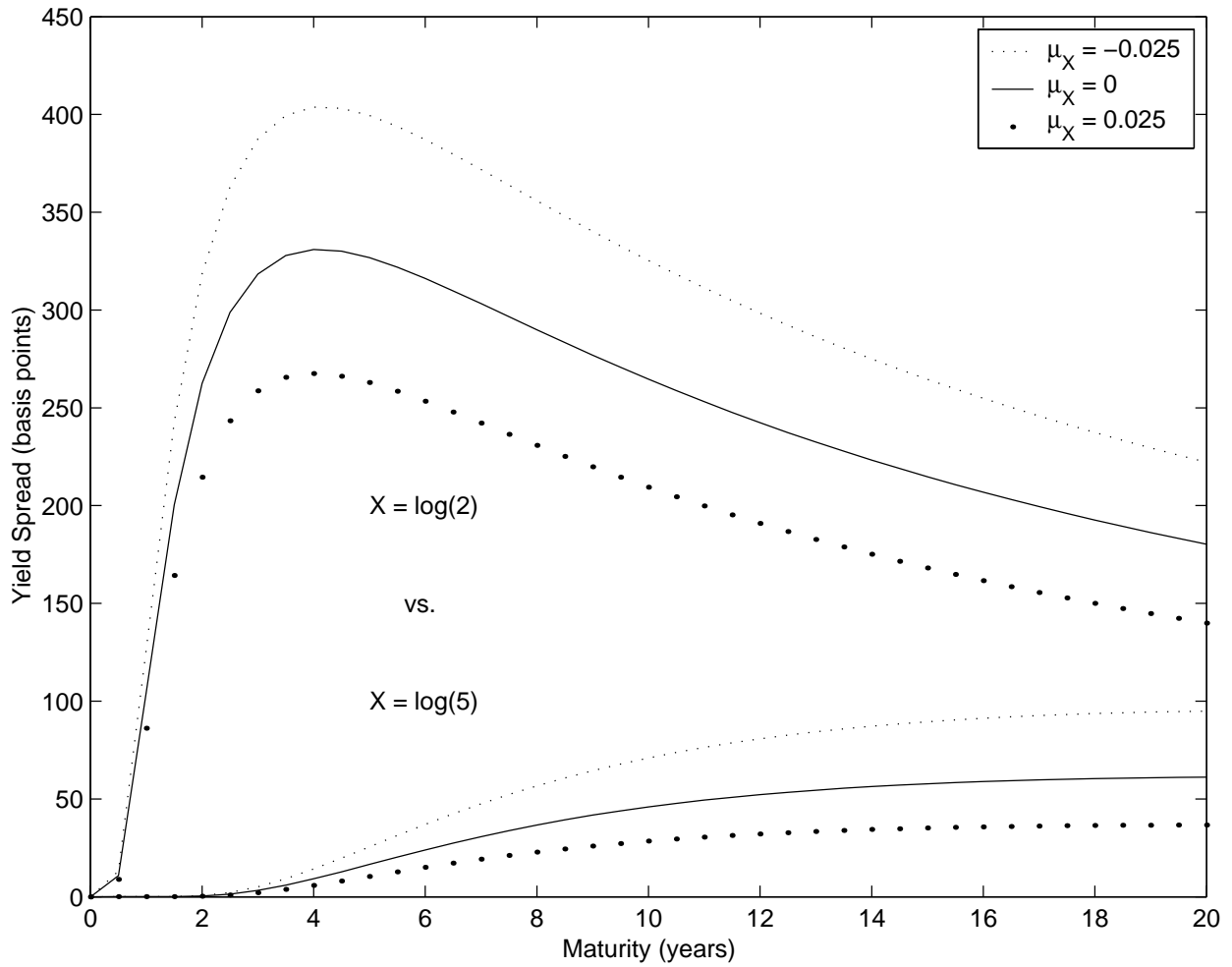


Figure 2. This figure plots the term structure of yield spread for different values of the risk neutral drift μ_X of the log-solvency ratio process at $X = \log(2)$ and $X = \log(5)$. The other parameters of the model are held fixed at $\rho_{Xr} = 0$, $\sigma_X = 0.3$, and $W = 0.5$.

Yield Spread for Different Values of ρ_{Xr}

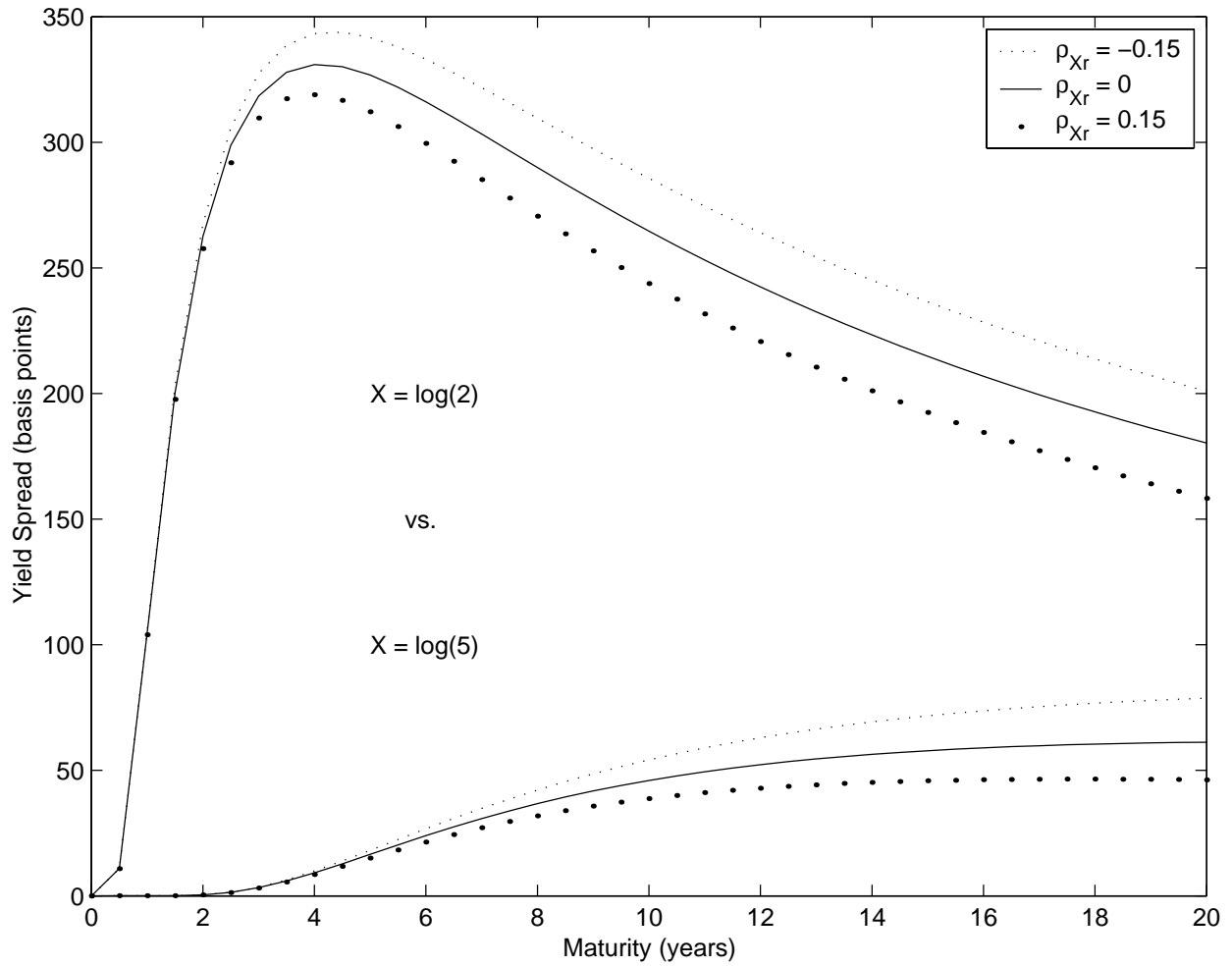


Figure 3. This figure plots the term structure of yield spread for different values of ρ_{Xr} at $X = \log(2)$ and $X = \log(5)$. The other parameters of the model are held fixed at $\mu_X = 0, \sigma_X = 0.3$, and $W = 0.5$.

Yield Spread for Different Values of σ_X

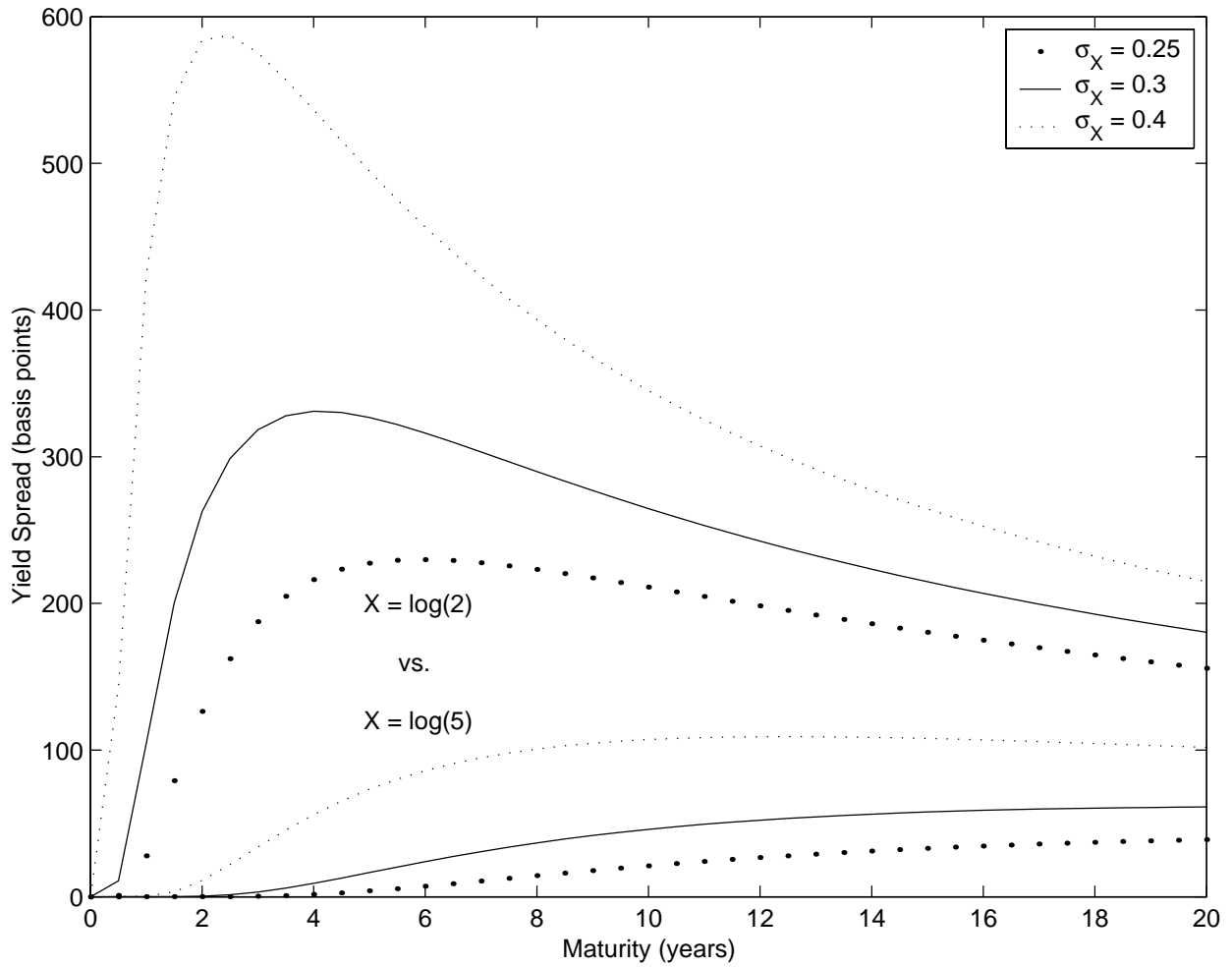


Figure 4. This figure plots the term structure of yield spread for different values of σ_X at $X = \log(2)$ and $X = \log(5)$. The other parameters of the model are held fixed at $\mu_X = 0, \rho_{Xr} = 0$, and $W = 0.5$.

Yield Spread for Different Values of W

